Interaction of Neutrons with Matter

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Overview

- Chain Reaction in a Nuclear Reactor
- Classification of Neutron Interactions
- Interaction Cross-Sections
- Neutron Flux and Reaction Rates
- Elastic Scattering: $X(n,n)X$ or simply $(n,n)$
- Inelastic Scattering: $(n,n')$
- $1/v$ and non-$1/v$ absorbers
Chain Reaction

\[ ^{235}\text{U} \rightarrow ^{239}\text{Pu} \]

Neutron Interactions

- Scattering
- Fission
- Capture
- Breeding

\[ \text{H}_2\text{O} \]

\[ E \sim 0.1 \text{ eV} \]

\[ n \rightarrow 2 \text{ MeV} \]

\[ 400 \text{ KeV} \]

\[ ^{238}\text{U} \rightarrow ^{239}\text{Pu} \]
Fission Cross Section

$^{235}$U

Capture

Fission
Physical Principles of Nuclear Reactor

\[ k \equiv \frac{N_2}{N_1} \]

- Fast fission
- Resonance abs.
- Non-fissile abs.
- Non-fuel abs.
- Leakage
- Slowing down
- Fission
- \( \nu \approx 2.5 \)
- \( 200 \text{ MeV/fission} \)

Neutron Interactions
Important

\[ \nu \approx 2.5 \text{ n/fiss} \]
\[ \epsilon_f \approx 200 \text{ MeV/fiss} \]
\[ \text{MeV} = 1.602 \times 10^{-13} \text{ J} \]

\[ P_{\text{thermal}} = N_{\text{fissions}} \cdot \epsilon_f \cdot \text{MeV} \]
\[ [W] = [\text{fiss/s}] \cdot [\text{MeV/fiss}] \cdot [\text{J/MeV}] \]

\( N_{\text{fissions}} \) may be evaluated through \( n(r,E,t) \)

\[ n(r,E,t)d^3r\,dE = \text{Number of neutrons in } d^3r \text{ about } r \text{ with energies in } dE \text{ about } E. \]
Conclusion

The behaviour of a nuclear reactor is governed by the distribution in

- space
- energy
- time

of the neutrons in the system. The central problem is to predict this
distribution, or equivalently to derive an equation for the neutron
density, \( n(r,E,t) \) or more generally, \( n(r,E,\Omega,t) \).

The first step to this end is to study the neutron interaction with matter.
Classification of Neutron Interaction with Matter

- Neutron Interaction
  - Scattering
    - Elastic
    - Inelastic
  - Absorption
    - Fission
    - Capture
      - $(n,\gamma)$
      - $(n,2n)$
      - $(n,3n)$
      - ...
    - N. mult.
      - $(n,p)$
      - $(n,\alpha)$
Reactions (n,xn)

\[ ^{63}_{29}\text{Cu} + ^{1}_{0}\text{n} \rightarrow ^{62}_{29}\text{Cu} + ^{1}_{0}\text{n} \]
\[ ^{63}_{29}\text{Cu} (n, 2n) ^{62}_{29}\text{Cu} \]

\[ ^{203}_{81}\text{Tl} + ^{1}_{0}\text{n} \rightarrow ^{200}_{81}\text{Tl} + ^{1}_{0}\text{n} \]
\[ ^{203}_{81}\text{Tl} (n, 4n) ^{200}_{81}\text{Tl} \]
Reactions \((n,p)\) \((n,d)\) \((n,\alpha)\)

\[ p \quad ^{14}_7\text{N} + ^1_0\text{n} \rightarrow ^{14}_6\text{C} + ^1_1\text{H} \quad ^{14}\text{C} \text{ production in the atmosphere} \]

\[ d \quad ^{31}_{15}\text{P} + ^1_0\text{n} \rightarrow ^{31}_{14}\text{Si} + ^2_1\text{H} \]

\[ \alpha \quad ^{10}_{5}\text{B} + ^1_0\text{n} \rightarrow \left( ^{11}_{5}\text{B} \right)^* \rightarrow ^7_3\text{Li} + ^4_2\text{He} \]

or \(^{10}_5\text{B}(n,\alpha)^7\text{Li}\)
Each instance of interaction will be called collision.

Area of the beam, $A$

Cross section, $\sigma$

Monoenergetic beam of neutrons

$[I] = \frac{\#}{\text{cm}^2 \text{s}}$

Volume, $\text{Vol} = A \times L$

Experiments:

$R_{\text{coll}}(\text{Vol}) = \sigma \times I \times N \times \text{Vol}$

$[\sigma] = \text{cm}^2$
It was observed in experiments that the fraction of neutrons which suffer their first collision in $dx$ around $x$ is constant.

$$-\frac{dI(x)}{I(x)} = dx \cdot \Sigma \quad [\text{cm}^{-1}]$$

---

**Neutrons have no memory**
Microscopic Cross Section

It was observed further in experiments that this constant is proportional to the density of the background medium.

\[ \Sigma = \sigma N \]

\( \sigma \left[ \text{cm}^2 \right] \)

Cross-sectional unit: 1 barn = 10^{-24} \text{ cm}^2

\[ \Sigma \rightarrow \Sigma_t \quad \text{(total cross-section)} \]

\[ \sigma \rightarrow \sigma_t \]
Example 1

A beam of 1-MeV neutrons of intensity $5 \times 10^8$ n/(cm$^2$s) strikes a thin $^{12}$C target. The area of the target is 0.5 cm$^2$ and is 0.05 cm thick. The beam has a cross-sectional area of 0.1 cm$^2$. At 1 MeV, the total cross-section of $^{12}$C is 2.6 b and density is 1.6 g/cm$^3$.

Question (a): At what rate do interactions take place in the target?

Question (b): What is the probability that a neutron in the beam will have a collision in the target?

$$R_{coll}(Vol) = \sigma \times I \times N \times Vol$$

Number density of the target material
Solution 1

\[ R_{\text{coll}}(Vol) = \sigma \times I \times N \times Vol \]

\[ N = \frac{\rho \cdot N_A}{M} = \frac{1.6 \times 6.02 \cdot 10^{23}}{12} \approx 8 \times 10^{22} \]

\[ R_{\text{coll}}(Vol) = 2.6 \times 10^{-24} \times 5 \times 10^5 \times 8 \times 10^{22} \times 0.1 \times 0.05 = 5.2 \times 10^5 \frac{\#}{s} \]

\[ \text{Pr}\{\text{neutron interacts}\} = \frac{R_{\text{coll}}}{I \times A} = \frac{5.2 \times 10^5}{5 \times 10^8 \times 0.1} = 1.04 \times 10^{-2} \]
Neutron Attenuation

\[-\frac{dI(x)}{I(x)} = \Sigma_t \quad I(x) = I(0)e^{-\Sigma_t x} \quad \Pr\{\text{No collision}\} = \frac{I(x)}{I(0)} = e^{-\Sigma_t x}\]

- Number of neutrons survived traveling distance \(x\).

\[-dI(x) \quad \text{Number of neutrons that survived penetrating the distance } x \text{ and made their first collision within } dx.\]

\[-\frac{dI(x)}{I(x)} = \text{Probability of the first collision within } dx = \Sigma_t dx\]

\[\Sigma_t = \text{Probability of some interaction per unit path length}\]
Probability of Uncollided Flight

Probability of uncollided flight within distance $x$

$$P(x) = e^{-\Sigma_t x}$$

Probability of at least one collision within distance $x$

$$F(x) = 1 - P(x) = 1 - e^{-\Sigma_t x}$$
First Collision Probability

Independent events: \[ P \{A \cap B\} = P \{A\} \cdot P \{B\} \]

Let \( p(x)dx \) be the probability that a neutron will have its first collision in \( dx \) around \( x \).

\[
p(x)dx = \Pr\{\text{(Survives } x\text{) \cdot (Collides in } dx\text{)}\} = \\
= \Pr\{\text{Survives } x\} \cdot \Pr\{\text{Collides in } dx\} = e^{-\Sigma x} \Sigma dx
\]

First collision probability: \[ p(x) = \Sigma e^{-\Sigma x} \] \[ \int_0^\infty p(x)dx = 1 \]
Mean Free Path

The mean free path, average distance traveled by a neutron in the medium is

\[ mfp = \lambda = \int_0^\infty xp(x)dx = \Sigma_t \int_0^\infty xe^{-\Sigma_t x}dx = \frac{1}{\Sigma_t} \]

For example, mean free path of 100-keV neutrons in liquid sodium is 11.6 cm.

\[ \sigma_t = 3.4 \text{ b}; \quad N = 2.54 \times 10^{22}; \quad \Sigma_t = 0.0864 \text{ cm}^{-1}; \quad \lambda = 1/\Sigma_t = 11.6 \text{ cm} \]
Mean Time

Collision probability per unit path length is \( \Sigma_t \Rightarrow \frac{1}{\Sigma_t} = \lambda_t \)  

Mean free path

Collision probability per unit time is \( \nu \Sigma_t \Rightarrow \frac{1}{\nu \Sigma_t} = \tau_t \)  

Mean time

Alternatively \( \frac{\lambda_t}{\nu} = \frac{1}{\nu \Sigma_t} = \tau_t \)
Homogeneous mixture of 2 nuclide species, $X$ and $Y$.

Probability per unit path length that a neutron collides with $X$ is $\Sigma_X = \sigma_X N_X$

Probability per unit path length that a neutron collides with $Y$ is $\Sigma_Y = \sigma_Y N_Y$

Total probability per unit path length that a neutron collides with either nuclides

$$\Sigma = \sigma_X N_X + \sigma_Y N_Y$$

It generalizes to

$$\Sigma = \sum\sigma_i N_i$$
Cross-Sections of Molecules

If there are $N$ molecules $X_m Y_n$ per cm$^3$ then $N_X = mN$, $N_Y = nN$

$$\sigma = \frac{\Sigma}{N} = \frac{\sigma_X N_X + \sigma_Y N_Y}{N} = m\sigma_X + n\sigma_Y$$

(except elastic scattering)

### Elastic scattering in water

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<td>$\sigma_s(H_2O)$</td>
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</table>
Elastic Scattering from Molecules

Fast neutron
Feels only H
\[ \tau_{\text{coll}} \sim 0 \]
\[ E_k \gg E_{\text{chem}} \]
\[ \sigma_s(\text{H}_2\text{O}) = 2\sigma_s(\text{H}) + \sigma_s(\text{O}) \]

Slow (thermal) neutron
Feels whole molecule
\[ \tau_{\text{coll}} \neq 0 \]
\[ E_k \sim E_{\text{chem}} \]
Elastic Scattering in Water

Chemical binding is

important unimportant
Neutron Flux

One-beam experiment:

\[ R_{\text{coll}}(\text{Vol}) = \sigma \times I \times N \times \text{Vol} \]

Collision rate:

\[ R_t \equiv \frac{R_{\text{coll}}(\text{Vol})}{\text{Vol}} = \Sigma_t I = \Sigma_t \nu n \]

Except for crystals, the interaction of neutrons with nuclei is independent of the angle.

\[ R_t = \Sigma_t (I_A + I_B + I_C + I_D) = \Sigma_t \nu (n_A + n_B + n_C + n_D) \]

\[ n = n_A + n_B + n_C + n_D \quad \rightarrow \quad R_t = \Sigma_t \nu n \]

\[ \phi \equiv \nu n \]

[\text{n/cm}^2\text{s}]
Meaning of cm$^2$ in Flux

$I = \nu n$

$\phi \equiv \nu n \left[ \frac{\#}{\text{cm}^2 \text{s}} \right]$
Physical Meaning of Neutron Flux

\[ \phi(r) \equiv \nu n(r) = \sum_i \nu n_i = \sum_i \phi_i \]

\[ \phi_i = \nu \frac{dN_i}{da} \]

\[ \phi = \nu \frac{dN}{da} \]

dN is the number of neutrons entering the sphere with a cross section \( da \) per unit time.

Flux is the number of neutrons entering the sphere with a unit cross section around point \( r \) per unit time.
Another Interpretation

\[ \phi(r) = v \cdot n(r) \]

distance traveled by 1 neutron per unit time

number of neutrons having velocity \( v \) in unit volume

Total distance (track length) traveled by all neutrons in unit volume around \( r \) per unit time
Reaction Rate

Reaction rate, \( R_x = \) number of interaction of kind \( x \) per unit volume in unit time

\[
R_x(r,t) = \sum_x \nu n(r,t) = \sum_x \phi(r,t)
\]

- Probability per unit path length
- Distance per unit time
- Probability per unit time
- Number of neutrons in unit volume
Neutron Flux Units

\[ \phi \equiv \nu \cdot n \]

\[ R_x = \Sigma_x \phi \]

\[ n(r, t) \left[ \frac{\#}{\text{cm}^3} \right] \Rightarrow \phi(r, t) \equiv \nu \cdot n(r, t) \left[ \frac{\#}{\text{s} \cdot \text{cm}^2} \right] \]

\[ n(r, E, t) \left[ \frac{\#}{\text{cm}^3 \cdot \text{J}} \right] \Rightarrow \phi(r, E, t) \equiv \nu \cdot n(r, E, t) \left[ \frac{\#}{\text{s} \cdot \text{cm}^2 \cdot \text{J}} \right] \]

\[ n(r, E, \Omega, t) \left[ \frac{\#}{\text{cm}^3 \cdot \text{J} \cdot \text{sr}} \right] \Rightarrow \phi(r, E, \Omega, t) \equiv \nu \cdot n(r, E, \Omega, t) \left[ \frac{\#}{\text{s} \cdot \text{cm}^2 \cdot \text{J} \cdot \text{sr}} \right] \]
Reactor Power

\[ P(t) = \varepsilon_f \cdot \int_0^\infty \int_0^\infty \Sigma_f (r, E)\phi(r, E, t) dE d^3r \]

\[ P(t) = \varepsilon_f \cdot \int_0^\infty \int_0^\infty \Sigma_f (r, E)vn(r, E, t) dE d^3r \]

\[
\begin{bmatrix} P \end{bmatrix} = \frac{J}{\#} \times \frac{1}{cm} \times \frac{\#}{s \times cm^2 \times J} \times J \times cm^3 = \frac{J}{s} = W
\]
Example 2

A certain research reactor of a cubic shape has a flux of $1 \times 10^{13}$ n/cm$^2$s and a side of 0.8 m.

If the fission cross-section, $\Sigma_f$, is 0.1 cm$^{-1}$, what is the power of the reactor?
Solution 2

\[ P = \varepsilon_f R_f V = \varepsilon_f \Sigma_f \phi V \]

\[ P = 200 \text{MeV} \quad 1.6 \times 10^{-13} \frac{\text{J}}{\text{MeV}} \times 0.1 \frac{1}{\text{cm}} \times 10^{13} \frac{\#}{\text{cm}^2 \text{s}} \times (80 \text{cm})^3 = 16.4 \text{ MW} \]

Typical NPP produces 1000 ÷ 3000 MW\(_{\text{th}}\)
Cross Section Hierarchy

Total cross section $\sigma_t$

- Scattering $\sigma_s$
  - Elastic $\sigma_e$
  - Inelastic $\sigma_i$

- Absorption $\sigma_a$
  - Fission $\sigma_f$
  - Capture $\sigma_c$
  - n. mult.
    - $\sigma_{2n}$
    - $\sigma_{3n}$
    - ...

- $\sigma_p$
- $\sigma_a$
Neutron Cross-Sections

\[ \sigma_s = \sigma_e + \sigma_i \]

\[ \sigma_a = \sigma_\gamma + \sigma_f + \sigma_p + \sigma_\alpha + \sigma_{2n} + \ldots \]

Often: \[ \sigma_a = \sigma_\gamma + \sigma_f \]

\[ \sigma_t = \sigma_s + \sigma_a \]
Compound Nucleus Formation

To explain nuclear reactions, Niels Bohr proposed in 1936 a two-stage model comprising the formation of a relatively long-lived intermediate nucleus and its subsequent decay.

\[
\frac{56}{26}\text{Fe} + n \rightarrow \left(\frac{57}{26}\text{Fe}\right)^* \rightarrow \frac{56}{26}\text{Fe} + n \quad \text{(elastic scattering)}
\]

\[
\frac{56}{26}\text{Fe} + n' \quad \text{(inelastic scattering)}
\]

\[
\frac{56}{26}\text{Fe} + \gamma \quad \text{(radiative capture)}
\]

\[
\frac{56}{26}\text{Fe} + 2n \quad \text{(n,2n) reaction}
\]

Cross-sections of nuclear reactions exhibit maxima at certain incident neutron energies. The maxima are called resonances.
Elastic Scattering

$X(n,n)X$

Regions:

- Potential scattering $\sigma_e = 4\pi R^2$
- Resonances
- Smooth region

Elastic scattering cross-section as a function of incident neutron energy, $E$
Elastic Scattering for $^{16}\text{O}$

\[ \sigma_e = 4\pi R^2 \]

- Resonance region
- Potential scattering
- ~50 keV
- Smooth region

Incident neutron data / ENDF/B-VI.8 / O16 / MT=2 : (z,z0) elastic scattering / Cross section

Cross-section (b)

Incident Energy (MeV)

- 0.01 eV
- 100 keV
Elastic Scattering for $^{238}\text{U}$

Incident neutron data / ENDF/B-V1.8 / U238 / MT=2 : ($z, z0$) elastic scattering / Cross section

Density increases

Incident Energy (MeV)

$\sim 10\text{ eV}$

1 eV

1 keV
Summary on Elastic Scattering

- Elastic scattering cross-sections for light elements are more or less independent of neutron energy up to 1 MeV.
- For intermediate and heavy elements, the elastic cross-section is constant at low energy and exhibit some variation at higher energy.
- We are usually more interested in light elements as far as elastic scattering is concerned; so a good approximation is, $\sigma_s = \text{const}$, for all elements of interest.
- Nearly all elements have scattering cross-sections in the range 2 to 20 barns.
- The important exception is water and heavy water.
Odd vs. Even Nuclides

Incident neutron data / ENDF/B-VII.0 / U238 / Resonances / Resonances data Resolved parameters : 1.0E-5 < E < 20000.0

- Incident Energy (eV)
- Resonances (b)
- 235U
- 238U

T=300.0 K

E

50 (eV)

0
Summary for Resonance Levels

• For light nuclei resonance levels are scarce with distance of about 100 keV or more. For heavy nuclei distance between the levels is of the order of few eV.

• Density of levels increase with energy. Pronounced effect of EVEN - UNEVEN nuclei. When neutrons interact with nuclei with EVEN mass number resonances are at larger distances than for UNEVEN mass numbers.
Inelastic Scattering

\[ {}^A X \rightarrow {}^{A+1} X^* \]

\[ n \rightarrow \theta, \gamma \]

(1) Light

1 MeV

Ground

Li, Be

2-3 MeV

(2) Heavy

50 KeV

\[ ^{238}U \]

40 KeV
General Conclusion on In. Sc.

- Inelastic scattering cross section is relatively small for light nuclei.
- Heavy nuclei cannot serve as good moderators in thermal reactors.
- Inelastic scattering may be significant:
  - Heterogeneous reactors
  - Highly enriched fuel
  - Fast reactors
Radiative Capture

The radiative capture cross-section as a function of the incident neutron energy

\[ \sigma_\gamma \propto \frac{1}{\sqrt{E}} \propto \frac{1}{v} \]

Potential scattering  \hspace{1cm} Resonances  \hspace{1cm} Smooth region

There are a few important nuclei that do not show exact $1/v$ behaviour. They are called non-$1/v$ absorbers.
Radiative Capture for $^{197}$Au

Incident neutron data / ENDF/B-VI.8 / Au197 / MT=102 : (z,g) radiative capture / Cross section

Slope is $-1/2$

1/$v$ region
Breit-Wigner Formula

\[ \sigma_\gamma(E) = \frac{\gamma_r^2 g}{4\pi} \frac{\Gamma_n \Gamma_\gamma}{\left( E - E_r \right)^2 + \Gamma^2/4} \]

- Wavelength of neutrons with energy \( E_r \)
- Statistical factor
- Neutron width
- Radiation width
- Total width, \( \Gamma = \Gamma_n + \Gamma_\gamma \)
Single-Level Resonance

Breit-Wigner formula:

\[ \sigma_x(E) = \frac{\sigma_{x,max}}{(E - E_r)^2 / (\Gamma/2)^2 + 1} \cdot \sqrt{\frac{E_r}{E}} \]

\[ \sigma_s(E) = \frac{\sigma_{s,max}}{(E - E_r)^2 / (\Gamma/2)^2 + 1} \]
Charged-Particles Reactions

As a rule, the (n,p), (n,α) are endothermic and do not occur below some threshold energy. Their cross-sections tend to be small, even above the threshold, especially for heavier nuclei.

However, there are some important exothermic reactions in light nuclei:

\[ {}^{16}\text{O}(n,p){}^{16}\text{N} \xrightarrow{\beta^- \text{7 sec}} {}^{16}\text{O} + \gamma \]

\[ E_n > 9 \text{ MeV} \]

\[ E_\gamma > 6\div7 \text{ MeV} \]

\[ {}^{10}\text{B}(n,\alpha){}^{7}\text{Li} \]

1/\nu absorber

\[ {}^{6}\text{Li}(n,\alpha){}^{3}\text{H} \]

1/\nu absorber

Tritium production
Cross-Section of \((n,\alpha)\) for \(^{10}\text{B}\)
Total Cross-Section

At low energy

\[ \sigma_t = 4\pi R^2 + \frac{C}{\sqrt{E}} \]

\[ ^{239}\text{Pu} \]

Incident neutron data / ENDF/B-VI.8 / Pu239 / / Cross section

- Total
- Fission
- Capture
- Elastic
- Inelastic

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Hydrogen and Deuterium

The nuclides, $^1\text{H}$ and $^2\text{H}$, interact differently

- No formation of compound nucleus.
- No resonances.
- No excited states.
- No inelastic scattering.

$\sigma_s$ is constant up to 10 keV.
$\sigma_\gamma$ is $1/\nu$ up to 1 MeV.
Cross-Sections for $^1\text{H}$

Both total and elastic scattering

Radiative capture, $\sigma_r$

1/$\nu$ region

Incident neutron data / ENDF/B-VI.8 / H1 // Cross section

0.1 eV

100 keV
Example 3

\[ \sigma_\gamma (E) = \frac{C}{\sqrt{E}} \rightarrow \frac{\sigma_\gamma (E)}{\sigma_\gamma (E_0)} = \sqrt{\frac{E_0}{E}} \rightarrow \sigma_\gamma (E) = \sigma_\gamma (E_0) \sqrt{\frac{E_0}{E}} \]

At \( E = 0.253 \) eV, \( \sigma_\gamma ^{(1H)} = 0.032 \) b. What is \( \sigma_\gamma \) at 1 eV?

\[ \sigma_\gamma (1\text{eV}) = 0.332 \times \sqrt{\frac{0.253}{1}} = 0.0528 \text{ b} \]
Polyenergetic Neutrons

We define \( n(E) \) such that \( n(E)dE \) is the number of neutrons per unit volume with energies in \( dE \) about \( E \).

Polyenergetic neutron beam
\[
dI(E) = \nu(E)n(E)dE
\]

Differential collision rate
\[
dR_t(E) = \sum \nu(E)n(E)dE
\]

Total collision rate
\[
R_t = \int_0^\infty \sum \nu(E)n(E)dE
\]

Absorption collision rate
\[
R_a = \int_0^\infty \sum \nu(E)n(E)dE
\]
1/$v$ Absorbers

$$\Sigma_a(E) \nu(E) = \Sigma_a(E_0) \nu_0$$

$$R_a = \int_0^\infty \Sigma_a(E) \nu(E) n(E) dE = \Sigma_a(E_0) \nu_0 \int_0^\infty n(E) dE = \Sigma_a(E_0) \nu_0 n$$

For $1/v$ absorbers, the absorption rate is independent of the energy distribution. Equivalently, the absorption rate is the same as that for a monoenergetic beam of neutrons with arbitrary energy $E_0$ and intensity $\nu_0 n$.

Standard practice: to specify $1/v$ X-sections at the single energy $E_0 = 0.0253$ eV corresponding to $\nu_0 = 2200$ m/s.

The values of X-sections at 0.0253 eV are loosely referred to as thermal cross-sections.

The quantity $\nu_0 n$ is called 2200 m/s flux and denoted as

$$\phi_0 \equiv \nu_0 n \quad \Rightarrow \quad R_a = \Sigma_a(E_0) \phi_0$$
Non-1/$v$ Absorbers

There are a comparatively few important non-1/$v$ absorbers.

\[ R_a = \int_0^\infty \Sigma_a(E) \nu(E) n(E) dE \]

It is dependent on the neutron energy distribution $n(E)$.

Assuming Maxwellian distribution of $n(E)$, C.H. Wescott computed $R_a$ numerically.

\[ R_a = g_a(T) \Sigma_a(E_0) \phi_0 \]

Non 1/$v$ factor  
Equilibrium temperature
### Non-1/\nu Factors

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<th>T °C</th>
<th>Cd</th>
<th>In</th>
<th>135Xe</th>
<th>149Sm</th>
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<td>0.9294</td>
<td>0.9208</td>
</tr>
<tr>
<td>600</td>
<td>2.9031</td>
<td>1.1522</td>
<td>1.0914</td>
<td>2.0852</td>
<td>1.0072</td>
<td>1.0128</td>
<td>0.9229</td>
<td>0.9108</td>
</tr>
<tr>
<td>800</td>
<td>3.0455</td>
<td>1.2123</td>
<td>0.9887</td>
<td>1.9246</td>
<td>1.0146</td>
<td>1.0201</td>
<td>0.9182</td>
<td>0.9036</td>
</tr>
<tr>
<td>1000</td>
<td>3.0599</td>
<td>1.2915</td>
<td>0.8858</td>
<td>1.7568</td>
<td>1.0226</td>
<td>1.0284</td>
<td>0.9118</td>
<td>0.8956</td>
</tr>
</tbody>
</table>

\[
R_a = g_a(T)\Sigma_a(E_0)\phi_0
\]

A small indium foil is placed in a reactor where 2200 m/s flux is $5\times10^{12}$ n/cm²s. The neutron density can be represented by a Maxwellian distribution with a temperature of 600 °C. At what rate are neutrons absorbed per cm³ in the foil? $N_{In} = 0.0383\times10^{24}$ and $\sigma_a(E_0) = 194$ b.

\[
R_a = 1.15\times7.43\text{ cm}^{-1}\times5\times10^{12}\text{ n/(cm}^2\text{s}) = 4.27\times10^{13}\text{ abs/(cm}^3\text{s})
\]

HT 2008
Neutron Interactions
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Summary

• Cross-Section Definition and Meaning
• Mean Free Path, Mean Time
• Flux Definition and its Meanings
• Flux Attenuation
The END
Classification of Neutron Sources

- Radioisotope neutron sources (small, portable, reliable, low-cost, no maintenance)
  - Fission source based on $^{252}$Cf (overwhelming favourite)
  - $(\alpha,n)$ sources
  - $(\gamma,n)$ sources

- Nuclear reactors

- Accelerator-based neutron sources
  - proton and deuterium bombardment
  - electron bombardment and photo-nuclear reactions
  - Spallation neutron sources
## NEUTRON SOURCES

<table>
<thead>
<tr>
<th>Source</th>
<th>Half-life</th>
<th>Reaction</th>
<th>Neutron yield (n×s⁻¹×g⁻¹)</th>
<th>Neutron energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{124}$Sb-Be</td>
<td>60.9 d</td>
<td>($\gamma$,n)</td>
<td>$2.7 \times 10^9$</td>
<td>0.024</td>
</tr>
<tr>
<td>stibium-antimony</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{140}$La-Be</td>
<td>40.2 h</td>
<td>($\gamma$,n)</td>
<td>$10^7$</td>
<td>2.0</td>
</tr>
<tr>
<td>$^{210}$Po-Be</td>
<td>138 d</td>
<td>($\alpha$,n)</td>
<td>$1.28 \times 10^{10}$</td>
<td>4.3</td>
</tr>
<tr>
<td>$^{241}$Am-Be</td>
<td>458 y</td>
<td>($\alpha$,n)</td>
<td>$1.0 \times 10^7$</td>
<td>~4</td>
</tr>
<tr>
<td>$^{226}$Ra-Be</td>
<td>1620 y</td>
<td>($\alpha$,n)</td>
<td>$1.3 \times 10^7$</td>
<td>~4</td>
</tr>
<tr>
<td>$^{227}$Ac-Be</td>
<td>21.8 y</td>
<td>($\alpha$,n)</td>
<td>$1.1 \times 10^9$</td>
<td>~4</td>
</tr>
<tr>
<td>$^{239}$Pu-Be</td>
<td>24400 y</td>
<td>($\alpha$,n)</td>
<td>$10^9$</td>
<td>~4</td>
</tr>
<tr>
<td>$^{228}$Th-Be</td>
<td>1.91 y</td>
<td>($\alpha$,n)</td>
<td>$1.7 \times 10^{10}$</td>
<td>~4</td>
</tr>
<tr>
<td>$^{252}$Cf</td>
<td>2.65 y</td>
<td>fission</td>
<td>$2.3 \times 10^{12}$</td>
<td>2.3</td>
</tr>
</tbody>
</table>
Californium Source

Channel 1:
\[ ^{252}_{98}\text{Cf} \rightarrow ^{252}_{98}\text{Cf} \rightarrow \alpha \ (6 \text{ MeV}) \]

Channel 2:
\[ ^{252}_{98}\text{Cf} \rightarrow 3.8 \text{ n} \ (185 \text{ MeV}) \]

\[ N \sim e^{-0.88E} \sinh(\sqrt{2E}) \]

Time of Flight Data
Plate Data

Relative number of neutrons per unit of energy

Energy (MeV)
Fission and Spallation

FISSION

SPALLATION

High energy fission

Evaporation of particles: neutrons, protons, deuterons, alpha particles
Neutron Discovery

• In 1930 W. Bothe and H. Becker in Germany: $\alpha \rightarrow$ light elements (Be, B, Li); unusually strong penetrating radiation was produced.

• In 1932 Irène Joliot-Curie and Frédéric Joliot in Paris: (unknown) radiation $\rightarrow$ paraffin or any other hydrogen containing compound it ejected protons of very high energy.

• Finally (later in 1932) the physicist James Chadwick in England performed a series of experiments showing that the gamma ray hypothesis was untenable.
**NEUTRON DISCOVERY**

Sir James Chadwick, 1932 (Nobel prize - 1935)

Mme I. Curie and F. Joliot had an idea: \( \frac{9}{4}\text{Be} + \frac{4}{2}\text{He} \rightarrow \frac{13}{6}\text{C} + \gamma \)

Chadwick guessed: \( \frac{9}{4}\text{Be} + \frac{4}{2}\text{He} \rightarrow \frac{12}{6}\text{C} + \frac{1}{0}\text{n} + \gamma \)
Mass Evaluation

\[ M - \text{nucleus mass} \]
\[ m - \text{neutron mass} \]
\[ v_0 - \text{neutron velocity before collision} \]
\[ v_1 - \text{neutron velocity after collision} \]
\[ V_1 - \text{nucleus velocity after collision} \]

Applying collision laws, Chadwick derived:

\[
\begin{align*}
\frac{1}{2} M V_1^2 &= \frac{1}{2} m (v_0^2 + v_1^2) \\
M (v_0 + v_1) &= MV_1
\end{align*}
\]

\[ V_1 = \frac{2m}{M + m} v_0 \]

\[ V_1 \text{ was determined for two types of recoil nuclei.} \]

Masses of \( ^{11}_5 B \) and \( ^{14}_7 N \) were known, so through the reaction

\[ ^{11}_5 B + ^4_2 \text{He} \rightarrow ^{14}_7 \text{N} + ^1_0 \text{n} \]

Chadwick evaluated \( m = 1.2 \ m_p \). Current measurement: \( m = 1.0014 \ m_p \)
### Neutron

<table>
<thead>
<tr>
<th>Composition</th>
<th>(udd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family</td>
<td>Fermion</td>
</tr>
<tr>
<td>Interaction</td>
<td>Gravity, Electromagnetic, Weak, Strong</td>
</tr>
<tr>
<td>Antiparticle</td>
<td>Antineutron</td>
</tr>
<tr>
<td>Discovered</td>
<td>J. Chadwick</td>
</tr>
<tr>
<td>Electric charge</td>
<td>$0 \ (q_n &lt; 4 \times 10^{-20} \text{ e})$</td>
</tr>
<tr>
<td>Magnetic mom.</td>
<td>$\mu_n = -1.04187564 \times 10^{-3} \mu_B$</td>
</tr>
<tr>
<td>Spin</td>
<td>$1/2$</td>
</tr>
</tbody>
</table>
| Decay        | $n \rightarrow p + e^- + \bar{\nu} + 1.29 \text{ MeV}$  
$\tau_{1/2} = 10.3 \text{ min}; \ \tau = 14.9 \text{ min}$ |
| Mass         | $m_n = 1.6749543 \times 10^{-27} \text{ kg}$  
$= 1.008665012 \text{ u}$  
$= 939.5731 \text{ MeV/c}^2$  
$= 1.0014 \times m_p = 1839 \times m_e$ |
NEUTRON SOURCES: Fission, Fusion, Spallation

<table>
<thead>
<tr>
<th>Nuclear Reaction</th>
<th>Energy (MeV)</th>
<th>Number of neutrons per particle or event</th>
<th>Heat deposition (MeV/n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (d,n)</td>
<td>0.2</td>
<td>$8 \times 10^{-5}$ n/d</td>
<td>2500</td>
</tr>
<tr>
<td>W (e,n)</td>
<td>35</td>
<td>$1.7 \times 10^{-2}$ n/e</td>
<td>2000</td>
</tr>
<tr>
<td>Be (d,n)</td>
<td>15</td>
<td>$1.2 \times 10^{-2}$ n/d</td>
<td>1200</td>
</tr>
<tr>
<td>Fission $^{235}$U (n,f)</td>
<td>2.2</td>
<td>2.5 n/fission</td>
<td>80</td>
</tr>
<tr>
<td>Fusion (T,d)</td>
<td>~1</td>
<td>1 n/fusion</td>
<td>17</td>
</tr>
<tr>
<td>Pb spallation</td>
<td>~ GeV</td>
<td>20 n/p</td>
<td>23</td>
</tr>
<tr>
<td>$^{235}$U spallation</td>
<td>~ GeV</td>
<td>40 n/p</td>
<td>50</td>
</tr>
</tbody>
</table>
Photo Neutron Sources

\[ ^2_1 \text{H} + \gamma \rightarrow ^1_1 \text{H} + ^1_0 \text{n} \quad Q = -2.226 \text{ MeV} \]

\[ ^9_4 \text{Be} + \gamma \rightarrow ^8_4 \text{Be} + ^1_0 \text{n} \quad Q = -1.666 \text{ MeV} \]

All other target particles have much higher binding energy.
Charged Particle Bombardment

These neutron sources are based on electrostatic accelerators or cyclotrons.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7\text{Li}(p,n)^7\text{Be}$</td>
<td>$^1\text{H}$</td>
</tr>
<tr>
<td>$^3\text{H}(d,n)^3\text{He}$</td>
<td>$^1\text{H}^+$ = p proton</td>
</tr>
<tr>
<td>$^2\text{H}(d,n)^3\text{He}$</td>
<td>$^2\text{H}^+$ = d deuteron</td>
</tr>
<tr>
<td>$^3\text{H}(d,n)^4\text{He}$</td>
<td>$^3\text{H}^+$ = t triton</td>
</tr>
<tr>
<td>$^1\text{H}(t,n)^3\text{He}$</td>
<td></td>
</tr>
<tr>
<td>$^1\text{H}(^7\text{Li},n)^7\text{Be}$</td>
<td></td>
</tr>
</tbody>
</table>
Nuclear Reactors: HFL (High Flux Reactor in Grenoble)

1. Core. 2 Heavy water reflector. 3 Light water pool. 4 Cold source. 5 Hot source. 6 Horizontal channel. 7 Concrete shield.

Thermal neutron spectrum of HFR
IBR-2 Reactor in Dubna (Russia)
IBR-2
Important

\[ n \approx 2.5 \text{ n/fiss} \]
\[ \varepsilon_f \approx 200 \text{ MeV/fiss} \]
\[ \text{MeV} = 1.602 \times 10^{-13} \text{ J} \]

\[ P_{\text{thermal}} = N_{\text{fissions}} \cdot \varepsilon_f \cdot \text{MeV} \]
\[ [W] = [#\text{fiss/s}] \cdot [\text{MeV/fiss}] \cdot [\text{J/MeV}] \]

\[ n(r,E,t)d^3r dE = \text{Number of neutrons in } d^3r \text{ about } r \text{ with energies in } dE \text{ about } E. \]

\[ P_{th} (t) = \varepsilon_f \int_0^\infty \int_V \sigma_f (r, E) N_F (r) v n (r, E, t) d^3r dE \]